

# Bounded VNDF Sampling for the Smith–GGX BRDF (Supplementary Document)

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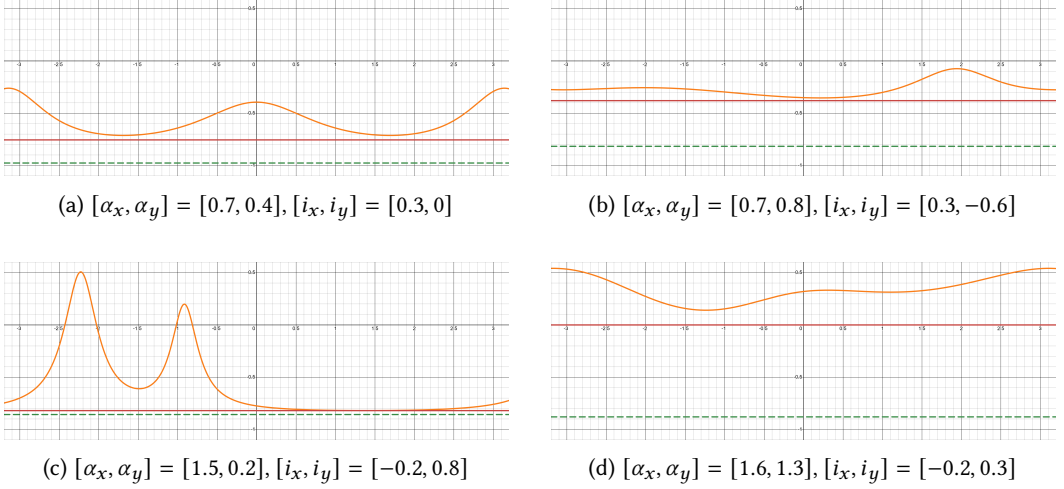


Fig. 1. Plots of the previous spherical cap (green dashed line), our spherical cap (red line), and the reflection vector bound projected into the unit-roughness space (orange line). The horizontal axis is the longitude  $\phi$  of the tangent-space reflection vector. The vertical axis is the cosine of the spherical cap angle (i.e.,  $\hat{o}_z$ ). Our spherical cap bounds the orange line more tightly than the previous spherical cap.

## 1 INTERACTIVE VISUALIZATION OF OUR LOWER BOUND

Fig. 1 shows plots of the previous and our spherical caps using different anisotropic roughness parameters and incoming directions. Online interactive graph is available at the following URL: <https://www.desmos.com/calculator/lpui8k1cky>.

## 2 KULLA AND CONTY’S MULTI-SCATTERING APPROXIMATION

Kulla and Conty [2017] approximated the multi-scattering term with a diffuse reflection model:

$$f_{\text{ms}}(\mathbf{i}, \mathbf{o}) \approx F_{\text{ms}} \frac{(1 - E(\mathbf{i})) (1 - E(\mathbf{o}))}{\pi (1 - E_{\text{avg}})} \chi^+(\mathbf{o} \cdot \mathbf{n}), \quad (1)$$

where  $E_{\text{avg}} = \int_{S^2} E(\boldsymbol{\omega}) |\boldsymbol{\omega} \cdot \mathbf{n}| / \pi d\boldsymbol{\omega}$  is the bi-hemispherical reflectance without the Fresnel term, and it is given by a lookup table or a fitted analytical approximation. For the multi-scattering Fresnel term, Hill [2018] found the following approximation:

$$F_{\text{ms}} \approx \frac{F_{\text{avg}}^2 E_{\text{avg}}}{1 - F_{\text{avg}}(1 - E_{\text{avg}})}, \quad (2)$$

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where  $F_{\text{avg}} = 2 \int_0^1 F(\mu)\mu d\mu$  is the average Fresnel term which can be approximated analytically [Kulla and Conty 2017]. Although this model is more expensive than Turquin's model [2019], it satisfies the reciprocity unlike Turquin's model.

## REFERENCES

- Stephen Hill. 2018. A Multi-Faceted Exploration (Part 2). <https://blog.selfshadow.com/2018/06/04/multi-faceted-part-2/>
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- Emmanuel Turquin. 2019. *Practical Multiple Scattering Compensation for Microfacet Models*. Technical Report. Industrial Light & Magic. [https://blog.selfshadow.com/publications/turquin/ms\\_comp\\_final.pdf](https://blog.selfshadow.com/publications/turquin/ms_comp_final.pdf)